Electromagnetic Fields & Waves

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Contents

1 Div, Grad, Curl, Laplacian

Divergence:

$$\text{Cartesian: } \nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\text{Cylindrical: } \nabla \cdot \mathbf{D} = \frac{1}{r}\frac{\partial}{\partial r}(rD_X) + \frac{1}{r}\frac{\partial D_\theta}{\partial \theta} + \frac{\partial D_z}{\partial z}$$

$$\text{Spherical: } \nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_X) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$$

Gradient:

$$\text{Cartesian: } \nabla V = \frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}$$

$$\text{Cylindrical: } \nabla V = \frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial \theta}a_\theta + \frac{\partial V}{\partial z}a_z$$

$$\text{Spherical: } \nabla V = \frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial \theta}a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}a_\phi$$

Curl:

$$\text{Cartesian: } \nabla \times \mathbf{H} = (\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z})a_x + (\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x})a_y + (\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y})a_z$$

$$\text{Cylindrical: } \nabla \times \mathbf{H} = (\frac{1}{r}\frac{\partial H_z}{\partial \theta} - \frac{\partial H_\theta}{\partial z})a_r + (\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r})a_\theta + \frac{1}{r}[\frac{\partial (rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}]a_z$$

Spherical:

$$\nabla \times \mathbf{H} = \frac{1}{r\sin\theta}[\frac{\partial H_z}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi})a_r + \frac{1}{r}[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial (rH_\phi)}{\partial r})a_\theta + \frac{1}{r}[\frac{\partial (rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}]a_\phi$$

Laplacian:

$$\text{Cartesian: } \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\text{Cylindrical: } \nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial V}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\text{Spherical: } \nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial V}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial V}{\partial \theta}) + \frac{1}{r^2 \sin\theta}\frac{\partial^2 V}{\partial \phi^2}$$

2 Vector identities

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0 \qquad \nabla \times (\nabla f) = 0 \qquad \nabla \cdot (\nabla f) = \nabla^2 f$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 f$$

$$\nabla(fg) = f\nabla g + g\nabla f \qquad \nabla \cdot (f\mathbf{G}) = \nabla f \cdot \mathbf{G} + f\nabla \cdot \mathbf{G}$$

$$\nabla \times (f\mathbf{G}) = \nabla f \times \mathbf{G} + f \times \nabla\mathbf{G}$$

$$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$$

$$\nabla \cdot)\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$$

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$$

3 Theorems

Divergence theorem:

$$\int_v \nabla \cdot \mathbf{A} \; dv = \int_s \mathbf{A} \cdot ds$$

Stoke's theorem:

$$\int_s \nabla \times \mathbf{A} \; ds = \int_c \mathbf{A} \cdot dl$$

Helmholtz' theorem:

A vector field is completely specified by its divergence and curl. Conversely, any vector field may be expressed as the sum of an irrotational vector and a solenoidal vector.

$$\text{Potential } \phi = -\int_{\infty}^{P} \mathbf{E} \cdot \mathbf{dl} \quad \text{(volts)}$$

Spherical conducting shell:

$$\text{Inside:} \quad E = 0 \quad \phi = \frac{Q}{4\pi\epsilon_0 R}$$

$$\text{Outside: } E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \phi = \frac{Q}{4\pi\epsilon_0 r}$$

(small r is outer radius of shell)

Charged wire (cylinder) of infinite length:

$$\phi_a - \phi_b = \frac{Q_l}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$$

Gauss' Law:

$$\oint_s \mathbf{D} \cdot \mathbf{ds} = \int_v Q_v dv \quad \text{or} \quad \nabla \cdot \mathbf{D} = Q_v$$

$$\text{Laplace: } \nabla^2 \phi = 0 \qquad \text{Poisson: } \nabla^2 \phi = -\frac{Q_v}{\epsilon}$$