Calculus

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1 Limits

1.

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \cos x = 1$$

2.

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log a (a > 0) \qquad \lim_{x \to 0} \frac{e^x - 1}{x} = \log e = 1$$

3. $\frac{1}{n} = \alpha$, then $\alpha \to 0$ as $n \to \infty$

$$\lim_{n \to \infty} (1 + \frac{k}{n})^n = \lim_{\alpha \to 0} (1 + k\alpha)^{1/\alpha} = e^k$$

If $k = 1$,

$$\lim_{n \to \infty} (1 + \frac{1}{n})^n = \lim_{\alpha \to 0} (1 + \alpha)^{1/\alpha} = e$$

4.

$$\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$$

5.

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

2 Derivatives

6. Provided the limit exists,

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

7. $\frac{\mathrm{d}k}{\mathrm{d}x} = 0$ where k is a constant

8. (In the following formulæ, consider this one:)

If y is a differentiable function of u and u is a differentiable function of x, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

9. Remember, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$f(x)$	$\frac{d}{dx} f(x)$	$f(u)(u = f(x))$	$\frac{d}{dx} f(u)$
x^n	nx^{n-1}	u^n	$nu^{n-1} \frac{du}{dx}$
$\sin x$	$\cos x$	$\sin u$	$\cos u \frac{du}{dx}$
$\cos x$	$-\sin x$	$\cos u$	$-\sin u \frac{du}{dx}$
$\tan x$	$\sec^2 x$	$\tan u$	$\sec^2 u \frac{du}{dx}$
$\cot x$	$-\text{cosec}^2 x$	$\cot u$	$-\text{cosec}^2 u \frac{du}{dx}$
$\sec x$	$\sec x \tan x$	$\sec u$	$\sec u \tan u \frac{du}{dx}$
$\text{cosec} x$	$-\text{cosec} x \cot x$	$\text{cosec} u$	$-\text{cosec} u \cot u \frac{du}{dx}$
e^x	e^x	e^u	$e^u \frac{du}{dx}$
a^x	$a^x \ln a$	a^u	$a^u \ln a \frac{du}{dx}$
$\ln x$	$1/x$	$\ln u$	$\frac{1}{u} \frac{du}{dx}$

10. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$ $\qquad$ $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

11. $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

$$\frac{d}{dx} \frac{u}{v} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad v \neq 0$$

12. If $y = f(x)$ is a differentiable function of x such that the inverse $x = g(y)$ is defined, then

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} \quad \left(\frac{dy}{dx} \neq 0\right)$$

13. If $x = g(y)$ is a differentiable function of y such that the inverse $y = f(x)$ is defined, then

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} \quad \left(\frac{dx}{dy} \neq 0\right)$$

2.0.1 Derivatives of inverse trigonometric functions